Math 3236 Statistical Theory

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Statistical Model.

Observations (have a random component).

$$\underset{\sim}{x} = (x_1, \ldots, x_N)$$

$$f(\underset{\sim}{x}; \underset{\sim}{\theta})$$

$$\underset{\sim}{\theta} \in \Omega \subset \mathbb{R}^n$$

Ω parameter space.

The observation are independent

$$f(\underset{\sim}{x}; \theta) = \prod_{i=1}^{N} f(x_i; \theta)$$

$$x_i \in \mathbb{R}^n$$

X_i is a random variable with p.d.f. $f(x_i ; \theta)$

$\underset{\sim}{X} = (X_i)$ is a random sample.

$f(x)$ To be a parameter for every x.

We will mostly look at parametric inferences:

$$f(x ; \underline{\theta})$$

Non-parametric inference.

$\underset{\sim}{\theta}$ are r.v. described by a p.d.f $\xi(\underline{\theta})$.

If we have a apriori dist on the parameter we then write

$$f(x \mid \underset{\sim}{\theta})$$

for the p.d.f. of X.

$$f_{\underset{\sim}{X}}(\underset{\sim}{x}) = \prod f(x_i \mid \underset{\sim}{\theta})\, \xi(\underset{\sim}{\theta})$$

$$L(\underset{\sim}{\theta} \mid \underset{\sim}{x}) = \prod_{i=1}^{N} f(x_i \mid \underset{\sim}{\theta})$$

Likelihood function.

Statistical Inference.

- find the value for θ?

Statistic: if $\underset{\sim}{X}$ is a sample $r(\underset{\sim}{X})$ where $r: \mathbb{R}^N \to \mathbb{R}$ is called "Statistic".

A statistic is called an "Estimator" for θ if

$r(\underline{X}) \to \theta$ in some sense

when $N \to \infty$

Consistent Estimator if

$r(\underline{X}) \xrightarrow{P} \theta$ $N \to \infty$

$\overline{X} = \frac{1}{N} \sum_i X_i$ This is an

estimator (consistent) for

$\mathbb{E}(X)$. If $\mathbb{E}(X) = m(\theta)$

To use the idea of the

method of moment I need

$m(\theta)$ to be invertible.

In such a case I can look

to $\mathbb{E}(X^2) = m_2(\theta)$

$$\overline{X^2} = \frac{1}{N} \sum_i X_i^2 \xrightarrow{p} m_2(\theta)$$

Prediction: Once I have a statement on the $\underline{\theta}$

$$f(x \mid \underline{\theta})$$

Find two statistics

$$r_1(\underline{X}) \qquad r_2(X)$$

such that

$$\mathbb{P}\left(r_2(X) \le \theta \le r_1(\underline{X})\right) = 0.95$$

Test of hypothesis.

h_0 : coin is fair

h_a : coin is biased against me

Bayesian Statistics

θ is a random variable with p.d.f. $\xi(\theta)$.

$\xi(\theta)$ is called the prior.

Ex: Bernoulli X_i it is reasonable to take

$\xi(\theta)$ uniform

on $\theta \in [0, 1]$.

The prior dist. is

$B(1, 1)$

If we observe a given value of x we can define the posterior dist

$$\xi(\theta | x) = \frac{f(x|\theta)\,\xi(\theta)}{\int_\Omega f(x|\theta)\,\xi(\theta)\,d\theta}$$

Probability of observing x (prior any observation) is

$$\int f(x|\theta)\,\xi(\theta)\,d\theta$$

$$\xi(\theta | x)$$

New extraction x_1

$$f(x_1 | x) = \int_\Omega f(x_1|\theta)\,\xi(\theta|x)\,d\theta$$

$$\xi(\theta | x, x_1) = \frac{f(x_1|\theta)\,\xi(\theta|x)}{f(x_1|x)}$$